From Vedic Square to the Digital Root 'Clockworks' of Modulo 90 Factorization

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	1	3	5	7	9
3	3	6	9	3	6	9	3	6	9
4	4	8	3	7	2	6	1	5	9
5	5	1	6	2	7	3	8	4	9
6	6	3	9	6	3	9	6	3	9
7	7	5	3	1	8	6	4	2	9
8	8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9	9

"Vedic Mathematics is...a sophisticated pedagogic and research tool ... ."
– Dr. L.M. Singhvi, former UK High Commissioner for India,
from his forward to *Cosmic Calculator*, by Williams & Gaskell

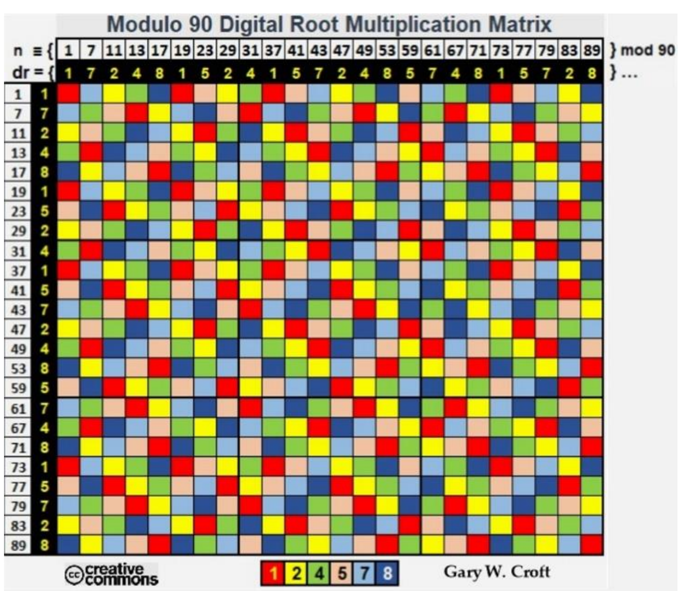
Mathematicians from India's ancient past are credited with inventing the 'Vedic Square' (pictured above). As anyone who has studied Vedic mathematics is aware, this deceptively simple multiplication matrix is a powerful analytical and computational tool.

The numbers in the table's column and row headings are cross-multiplied, and their products converted to their digital roots, distributed across 9^2 cells. [Digital root (aka repeated digit sum, iterated sum-of-digits, or modulus 9 function) refers to the single digit (1 - 9) that results when all digit sums are summed in turn; this reflects the fact that any natural number is congruent modulo 9 to its digital root. For example: 89 = 8+9 = 17 = 1+7 = 8, or dr(89) = 8.]

When it comes to The Digital Root, we completely agree with the importance attributed to it by Talal Ghannam, Ph.D. (in physics): "It was through my search for meaning that I discovered how important the digital root space is, which in return rewarded me with a look at how elegant and beautiful the world really is." from *The Mystery of Numbers Revealed Through Their Digital Root.*

In the color-coded matrix below we apply the Vedic Square *concept* to the period-24 digital root of natural numbers not divisible by 2, 3, or 5–otherwise known as **n ≡ {1, 7, 11, 13, 17, 19, 23, 29} modulo 30**. We then expand the frame (x 3) to encompass **n ≡ {1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89} modulo 90**. We'll call this sequence our 'domain.' This action synchronizes the modulus with its period-24 digital root, and – as we shall discover – creates an explosion of symmetries.

By definition, our domain consists of the number 1 and all prime numbers > 5 and their multiplicative multiples. The matrix below colorfully exposes the perfect symmetry underpinning factorization of our domain at the digital root level. This object expands geometrically to infinity, and can be configured to spiral within a modulo 90 factorization wheel (ref: primesdemystified.com):



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Given that by definition none of the elements in our defined domain are divisible by 3, you'll find no 3's, 6's, or 9's in its period-24 digital root sequence, viz. **{1, 7, 2, 4, 8, 1, 5, 2, 4, 1, 5, 7, 2, 4, 8, 5, 7, 4, 8, 1, 5, 7, 2, 8} {repeat …}**, comprised 4 each of numbers 1, 2, 4, 5, 7, and 8. This knowledge allows us to 'collapse' the Vedic Square, as pictured below (and we note that technically the subset {1, 2, 4, 5, 7, 8} forms a cyclic group of multiplicative units in the ring $\mathbb{Z}/9\mathbb{Z}$.)

	1	2	4	5	7	8
1	1	2	4	5	7	8
2	2	4	8	1	5	7
4	4	8	7	2	1	5
5	5	1	2	7	8	4
7	7	5	1	8	4	2
8	8	7	5	4	2	1

From our 'Vedic Square Light' matrix we can then extrapolate the 36 digital root dyads that drive factorization sequencing of composite numbers within our domain:

The 36 Fundamental Digital Root Multiplication Dyads

For all composite numbers congruent to {1, 7, 11, 13, 17, 19, 23, 29} modulo 30*

1 →	1 x 1 = 1 →	2 x 5 = 1 →	4 x 7 = 1 →	5 x 2 = 1 →	7 x 4 = 1 →	8 x 8 = 1
2 →	1 x 2 = 2 →	2 x 1 = 2 →	4 x 5 = 2 →	5 x 4 = 2 →	7 x 8 = 2 →	8 x 7 = 2
4 →	1 x 4 = 4 →	2 x 2 = 4 →	4 x 1 = 4 →	5 x 8 = 4 →	7 x 7 = 4 →	8 x 5 = 4
5 →	1 x 5 = 5 →	2 x 7 = 5 →	4 x 8 = 5 →	5 x 1 = 5 →	7 x 2 = 5 →	8 x 4 = 5
7 →	1 x 7 = 7 →	2 x 8 = 7 →	4 x 4 = 7 →	5 x 5 = 7 →	7 x 1 = 7 →	8 x 2 = 7
8 →	1 x 8 = 8 →	2 x 4 = 8 →	4 x 2 = 8 →	5 x 7 = 8 →	7 x 5 = 8 →	8 x 1 = 8

*Also defined as numbers not divisible by 2, 3 or 5.

Then, again employing the Vedic Square *concept*, we produce a sister matrix, albeit in her case we've converted the cross-multiplied products to their *modulo 90 congruence* rather than their digital roots (and note that the principal diagonal (highlighted in blue) sequences the squares, while the secondary diagonal (highlighted in pink) is comprised exclusively of prime numbers):

Modulo 90 Factorization Congruency

Period-24 Factorization Matrix for Composite Numbers Congruent to ...

n ≡ {1,7,11,13,17,19,23,29,31,37,41,43,47,49,53,59,61,67,71,73,77,79,83,89} modulo 90

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
		1	**7**	**11**	**13**	**17**	**19**	**23**	**29**	**31**	**37**	**41**	**43**	**47**	**49**	**53**	**59**	**61**	**67**	**71**	**73**	**77**	**79**	**83**	**89**
1	**1**	1	7	11	13	17	19	23	29	31	37	41	43	47	49	53	59	61	67	71	73	77	79	83	89
2	**7**	7	49	77	1	29	43	71	23	37	79	17	31	59	73	11	53	67	19	47	61	89	13	41	83
3	**11**	11	77	31	53	7	29	73	49	71	47	1	23	67	89	43	19	41	17	61	83	37	59	13	79
4	**13**	13	1	53	79	41	67	29	17	43	31	83	19	71	7	59	47	73	61	23	49	11	37	89	77
5	**17**	17	29	7	41	19	53	31	43	77	89	67	11	79	23	1	13	47	59	37	71	49	83	61	73
6	**19**	19	43	29	67	53	1	77	11	49	73	59	7	83	31	17	41	79	13	89	37	23	61	47	71
7	**23**	23	71	73	29	31	77	79	37	83	41	43	89	1	47	49	7	53	11	13	59	61	17	19	67
8	**29**	29	23	49	17	43	11	37	31	89	83	19	77	13	71	7	1	59	53	79	47	73	41	67	61
9	**31**	31	37	71	43	77	49	83	89	61	67	11	73	17	79	23	29	1	7	41	13	47	19	53	59
10	**37**	37	79	47	31	89	73	41	83	67	19	77	61	29	13	71	23	7	49	17	1	59	43	11	53
11	**41**	41	17	1	83	67	59	43	19	11	77	61	53	37	29	13	79	71	47	31	23	7	89	73	49
12	**43**	43	31	23	19	11	7	89	77	73	61	53	49	41	37	29	17	13	1	83	79	71	67	59	47
13	**47**	47	59	67	71	79	83	1	13	17	29	37	41	49	53	61	73	77	89	7	11	19	23	31	43
14	**49**	49	73	89	7	23	31	47	71	79	13	29	37	53	61	77	11	19	43	59	67	83	1	17	41
15	**53**	53	11	43	59	1	17	49	7	23	71	13	29	61	77	19	67	83	41	73	89	31	47	79	37
16	**59**	59	53	19	47	13	41	7	1	29	23	79	17	73	11	67	61	89	83	49	77	43	71	37	31
17	**61**	61	67	41	73	47	79	53	59	1	7	71	13	77	19	83	89	31	37	11	43	17	49	23	29
18	**67**	67	19	17	61	59	13	11	53	7	49	47	1	89	43	41	83	37	79	77	31	29	73	71	23
19	**71**	71	47	61	23	37	89	13	79	41	17	31	83	7	59	73	49	11	77	1	53	67	29	43	19
20	**73**	73	61	83	49	71	37	59	47	13	1	23	79	11	67	89	77	43	31	53	19	41	7	29	17
21	**77**	77	89	37	11	49	23	61	73	47	59	7	71	19	83	31	43	17	29	67	41	79	53	1	13
22	**79**	79	13	59	37	83	61	17	41	19	43	89	67	23	1	47	71	49	73	29	7	53	31	77	11
23	**83**	83	41	13	89	61	47	19	67	53	11	73	59	31	17	79	37	23	71	43	29	1	77	49	7
24	**89**	89	83	79	77	73	71	67	61	59	53	49	47	43	41	37	31	29	23	19	17	13	11	7	1

The additive sum of all columns and rows = 1080 = 360 x 3. All diagonals are palindromic.

Matrix is the source of 24 x 24 = 576 factorization dyads

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And the beauty of it is that from the matrix pictured above we can extrapolate 24 sets of 24 = 576 modulo 90 factorization dyads that account for all composite numbers within our domain (and note that each set has perfect bilateral 90-sum symmetry):

24 x 24 = 576 Modulo 90 Factorization Dyads for Composite Numbers Congruent to {1,7,11,13,17,19,23,29,31,37,41,43,47,49,53,59,61,67,71,73,77,79,83,89} Modulo 90

(all 24 sets of 24 dyads have vertical bilateral 90-sum symmetry)

n ≡ {1} Mod 90	n ≡ {7} Mod 90	n ≡ {11} Mod 90	n ≡ {13} Mod 90	n ≡ {17} Mod 90	n ≡ {19} Mod 90
n ≡ {1} x {1}	n ≡ {1} x {7}	n ≡ {1} x {11}	n ≡ {1} x {13}	n ≡ {1} x {17}	n ≡ {1} x {19}
n ≡ {7} x {13}	n ≡ {7} x {1}	n ≡ {7} x {53}	n ≡ {7} x {79}	n ≡ {7} x {41}	n ≡ {7} x {67}
n ≡ {11} x {41}	n ≡ {11} x {17}	n ≡ {11} x {1}	n ≡ {11} x {83}	n ≡ {11} x {67}	n ≡ {11} x {59}
n ≡ {13} x {7}	n ≡ {13} x {49}	n ≡ {13} x {77}	n ≡ {13} x {1}	n ≡ {13} x {29}	n ≡ {13} x {43}
n ≡ {17} x {53}	n ≡ {17} x {11}	n ≡ {17} x {43}	n ≡ {17} x {59}	n ≡ {17} x {1}	n ≡ {17} x {17}
n ≡ {19} x {19}	n ≡ {19} x {43}	n ≡ {19} x {29}	n ≡ {19} x {67}	n ≡ {19} x {53}	n ≡ {19} x {1}
n ≡ {23} x {47}	n ≡ {23} x {59}	n ≡ {23} x {67}	n ≡ {23} x {71}	n ≡ {23} x {79}	n ≡ {23} x {83}
n ≡ {29} x {59}	n ≡ {29} x {53}	n ≡ {29} x {19}	n ≡ {29} x {47}	n ≡ {29} x {13}	n ≡ {29} x {41}
n ≡ {31} x {61}	n ≡ {31} x {67}	n ≡ {31} x {41}	n ≡ {31} x {73}	n ≡ {31} x {47}	n ≡ {31} x {79}
n ≡ {37} x {73}	n ≡ {37} x {61}	n ≡ {37} x {83}	n ≡ {37} x {49}	n ≡ {37} x {71}	n ≡ {37} x {37}
n ≡ {41} x {11}	n ≡ {41} x {77}	n ≡ {41} x {31}	n ≡ {41} x {53}	n ≡ {41} x {7}	n ≡ {41} x {29}
n ≡ {43} x {67}	n ≡ {43} x {19}	n ≡ {43} x {17}	n ≡ {43} x {61}	n ≡ {43} x {59}	n ≡ {43} x {13}
n ≡ {47} x {23}	n ≡ {47} x {71}	n ≡ {47} x {73}	n ≡ {47} x {29}	n ≡ {47} x {31}	n ≡ {47} x {77}
n ≡ {49} x {79}	n ≡ {49} x {13}	n ≡ {49} x {59}	n ≡ {49} x {37}	n ≡ {49} x {83}	n ≡ {49} x {61}
n ≡ {53} x {17}	n ≡ {53} x {29}	n ≡ {53} x {7}	n ≡ {53} x {41}	n ≡ {53} x {19}	n ≡ {53} x {53}
n ≡ {59} x {29}	n ≡ {59} x {23}	n ≡ {59} x {49}	n ≡ {59} x {17}	n ≡ {59} x {43}	n ≡ {59} x {11}
n ≡ {61} x {31}	n ≡ {61} x {37}	n ≡ {61} x {71}	n ≡ {61} x {43}	n ≡ {61} x {77}	n ≡ {61} x {49}
n ≡ {67} x {43}	n ≡ {67} x {31}	n ≡ {67} x {23}	n ≡ {67} x {19}	n ≡ {67} x {11}	n ≡ {67} x {7}
n ≡ {71} x {71}	n ≡ {71} x {47}	n ≡ {71} x {61}	n ≡ {71} x {23}	n ≡ {71} x {37}	n ≡ {71} x {89}
n ≡ {73} x {37}	n ≡ {73} x {79}	n ≡ {73} x {47}	n ≡ {73} x {31}	n ≡ {73} x {89}	n ≡ {73} x {73}
n ≡ {77} x {83}	n ≡ {77} x {41}	n ≡ {77} x {13}	n ≡ {77} x {89}	n ≡ {77} x {61}	n ≡ {77} x {47}
n ≡ {79} x {49}	n ≡ {79} x {73}	n ≡ {79} x {89}	n ≡ {79} x {7}	n ≡ {79} x {23}	n ≡ {79} x {31}
n ≡ {83} x {77}	n ≡ {83} x {89}	n ≡ {83} x {37}	n ≡ {83} x {11}	n ≡ {83} x {49}	n ≡ {83} x {23}
n ≡ {89} x {89}	n ≡ {89} x {83}	n ≡ {89} x {79}	n ≡ {89} x {77}	n ≡ {89} x {73}	n ≡ {89} x {71}

n ≡ {23} Mod 90	n ≡ {29} Mod 90	n ≡ {31} Mod 90	n ≡ {37} Mod 90	n ≡ {41} Mod 90	n ≡ {43} Mod 90
n ≡ {1} x {23}	n ≡ {1} x {29}	n ≡ {1} x {31}	n ≡ {1} x {37}	n ≡ {1} x {41}	n ≡ {1} x {43}
n ≡ {7} x {29}	n ≡ {7} x {17}	n ≡ {7} x {43}	n ≡ {7} x {31}	n ≡ {7} x {83}	n ≡ {7} x {19}
n ≡ {11} x {43}	n ≡ {11} x {19}	n ≡ {11} x {11}	n ≡ {11} x {77}	n ≡ {11} x {61}	n ≡ {11} x {53}
n ≡ {13} x {71}	n ≡ {13} x {23}	n ≡ {13} x {37}	n ≡ {13} x {79}	n ≡ {13} x {17}	n ≡ {13} x {31}
n ≡ {17} x {49}	n ≡ {17} x {7}	n ≡ {17} x {23}	n ≡ {17} x {71}	n ≡ {17} x {13}	n ≡ {17} x {29}
n ≡ {19} x {77}	n ≡ {19} x {11}	n ≡ {19} x {49}	n ≡ {19} x {73}	n ≡ {19} x {59}	n ≡ {19} x {7}
n ≡ {23} x {1}	n ≡ {23} x {13}	n ≡ {23} x {17}	n ≡ {23} x {29}	n ≡ {23} x {37}	n ≡ {23} x {41}
n ≡ {29} x {7}	n ≡ {29} x {1}	n ≡ {29} x {29}	n ≡ {29} x {23}	n ≡ {29} x {79}	n ≡ {29} x {17}
n ≡ {31} x {53}	n ≡ {31} x {59}	n ≡ {31} x {1}	n ≡ {31} x {7}	n ≡ {31} x {71}	n ≡ {31} x {13}
n ≡ {37} x {59}	n ≡ {37} x {47}	n ≡ {37} x {13}	n ≡ {37} x {1}	n ≡ {37} x {23}	n ≡ {37} x {79}
n ≡ {41} x {73}	n ≡ {41} x {49}	n ≡ {41} x {71}	n ≡ {41} x {47}	n ≡ {41} x {1}	n ≡ {41} x {23}
n ≡ {43} x {11}	n ≡ {43} x {53}	n ≡ {43} x {7}	n ≡ {43} x {49}	n ≡ {43} x {47}	n ≡ {43} x {1}
n ≡ {47} x {79}	n ≡ {47} x {37}	n ≡ {47} x {83}	n ≡ {47} x {41}	n ≡ {47} x {43}	n ≡ {47} x {89}
n ≡ {49} x {17}	n ≡ {49} x {41}	n ≡ {49} x {19}	n ≡ {49} x {43}	n ≡ {49} x {89}	n ≡ {49} x {67}
n ≡ {53} x {31}	n ≡ {53} x {43}	n ≡ {53} x {77}	n ≡ {53} x {89}	n ≡ {53} x {67}	n ≡ {53} x {11}
n ≡ {59} x {37}	n ≡ {59} x {31}	n ≡ {59} x {89}	n ≡ {59} x {83}	n ≡ {59} x {19}	n ≡ {59} x {77}
n ≡ {61} x {83}	n ≡ {61} x {89}	n ≡ {61} x {61}	n ≡ {61} x {67}	n ≡ {61} x {11}	n ≡ {61} x {73}
n ≡ {67} x {89}	n ≡ {67} x {77}	n ≡ {67} x {73}	n ≡ {67} x {61}	n ≡ {67} x {53}	n ≡ {67} x {49}
n ≡ {71} x {13}	n ≡ {71} x {79}	n ≡ {71} x {41}	n ≡ {71} x {17}	n ≡ {71} x {31}	n ≡ {71} x {83}
n ≡ {73} x {41}	n ≡ {73} x {83}	n ≡ {73} x {67}	n ≡ {73} x {19}	n ≡ {73} x {77}	n ≡ {73} x {61}
n ≡ {77} x {19}	n ≡ {77} x {67}	n ≡ {77} x {53}	n ≡ {77} x {11}	n ≡ {77} x {73}	n ≡ {77} x {59}
n ≡ {79} x {47}	n ≡ {79} x {71}	n ≡ {79} x {79}	n ≡ {79} x {13}	n ≡ {79} x {29}	n ≡ {79} x {37}
n ≡ {83} x {61}	n ≡ {83} x {73}	n ≡ {83} x {47}	n ≡ {83} x {59}	n ≡ {83} x {7}	n ≡ {83} x {71}
n ≡ {89} x {67}	n ≡ {89} x {61}	n ≡ {89} x {59}	n ≡ {89} x {53}	n ≡ {89} x {49}	n ≡ {89} x {47}

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n ≡ {47} Mod 90	n ≡ {49} Mod 90	n ≡ {53} Mod 90	n ≡ {59} Mod 90	n ≡ {61} Mod 90	n ≡ {67} Mod 90
n ≡ {1} x {47}	n ≡ {1} x {49}	n ≡ {1} x {53}	n ≡ {1} x {59}	n ≡ {1} x {61}	n ≡ {1} x {67}
n ≡ {7} x {71}	n ≡ {7} x {7}	n ≡ {7} x {59}	n ≡ {7} x {47}	n ≡ {7} x {73}	n ≡ {7} x {61}
n ≡ {11} x {37}	n ≡ {11} x {29}	n ≡ {11} x {13}	n ≡ {11} x {79}	n ≡ {11} x {71}	n ≡ {11} x {47}
n ≡ {13} x {59}	n ≡ {13} x {73}	n ≡ {13} x {11}	n ≡ {13} x {53}	n ≡ {13} x {67}	n ≡ {13} x {19}
n ≡ {17} x {61}	n ≡ {17} x {77}	n ≡ {17} x {19}	n ≡ {17} x {67}	n ≡ {17} x {83}	n ≡ {17} x {41}
n ≡ {19} x {83}	n ≡ {19} x {31}	n ≡ {19} x {17}	n ≡ {19} x {41}	n ≡ {19} x {79}	n ≡ {19} x {13}
n ≡ {23} x {49}	n ≡ {23} x {53}	n ≡ {23} x {61}	n ≡ {23} x {73}	n ≡ {23} x {77}	n ≡ {23} x {89}
n ≡ {29} x {73}	n ≡ {29} x {11}	n ≡ {29} x {67}	n ≡ {29} x {61}	n ≡ {29} x {89}	n ≡ {29} x {83}
n ≡ {31} x {77}	n ≡ {31} x {19}	n ≡ {31} x {83}	n ≡ {31} x {89}	n ≡ {31} x {31}	n ≡ {31} x {37}
n ≡ {37} x {11}	n ≡ {37} x {67}	n ≡ {37} x {89}	n ≡ {37} x {77}	n ≡ {37} x {43}	n ≡ {37} x {31}
n ≡ {41} x {67}	n ≡ {41} x {89}	n ≡ {41} x {43}	n ≡ {41} x {19}	n ≡ {41} x {41}	n ≡ {41} x {17}
n ≡ {43} x {89}	n ≡ {43} x {43}	n ≡ {43} x {41}	n ≡ {43} x {83}	n ≡ {43} x {37}	n ≡ {43} x {79}
n ≡ {47} x {1}	n ≡ {47} x {47}	n ≡ {47} x {49}	n ≡ {47} x {7}	n ≡ {47} x {53}	n ≡ {47} x {11}
n ≡ {49} x {23}	n ≡ {49} x {1}	n ≡ {49} x {47}	n ≡ {49} x {71}	n ≡ {49} x {49}	n ≡ {49} x {73}
n ≡ {53} x {79}	n ≡ {53} x {23}	n ≡ {53} x {1}	n ≡ {53} x {13}	n ≡ {53} x {47}	n ≡ {53} x {59}
n ≡ {59} x {13}	n ≡ {59} x {71}	n ≡ {59} x {7}	n ≡ {59} x {1}	n ≡ {59} x {59}	n ≡ {59} x {53}
n ≡ {61} x {17}	n ≡ {61} x {79}	n ≡ {61} x {23}	n ≡ {61} x {29}	n ≡ {61} x {1}	n ≡ {61} x {7}
n ≡ {67} x {41}	n ≡ {67} x {37}	n ≡ {67} x {29}	n ≡ {67} x {17}	n ≡ {67} x {13}	n ≡ {67} x {1}
n ≡ {71} x {7}	n ≡ {71} x {59}	n ≡ {71} x {73}	n ≡ {71} x {49}	n ≡ {71} x {11}	n ≡ {71} x {77}
n ≡ {73} x {29}	n ≡ {73} x {13}	n ≡ {73} x {71}	n ≡ {73} x {23}	n ≡ {73} x {7}	n ≡ {73} x {49}
n ≡ {77} x {31}	n ≡ {77} x {17}	n ≡ {77} x {79}	n ≡ {77} x {37}	n ≡ {77} x {23}	n ≡ {77} x {71}
n ≡ {79} x {53}	n ≡ {79} x {61}	n ≡ {79} x {77}	n ≡ {79} x {11}	n ≡ {79} x {19}	n ≡ {79} x {43}
n ≡ {83} x {19}	n ≡ {83} x {83}	n ≡ {83} x {31}	n ≡ {83} x {43}	n ≡ {83} x {17}	n ≡ {83} x {29}
n ≡ {89} x {43}	n ≡ {89} x {41}	n ≡ {89} x {37}	n ≡ {89} x {31}	n ≡ {89} x {29}	n ≡ {89} x {23}

n ≡ {71} Mod 90	n ≡ {73} Mod 90	n ≡ {77} Mod 90	n ≡ {79} Mod 90	n ≡ {83} Mod 90	n ≡ {89} Mod 90
n ≡ {1} x {71}	n ≡ {1} x {73}	n ≡ {1} x {77}	n ≡ {1} x {79}	n ≡ {1} x {83}	n ≡ {1} x {89}
n ≡ {7} x {23}	n ≡ {7} x {49}	n ≡ {7} x {11}	n ≡ {7} x {37}	n ≡ {7} x {89}	n ≡ {7} x {77}
n ≡ {11} x {31}	n ≡ {11} x {23}	n ≡ {11} x {7}	n ≡ {11} x {89}	n ≡ {11} x {73}	n ≡ {11} x {49}
n ≡ {13} x {47}	n ≡ {13} x {61}	n ≡ {13} x {89}	n ≡ {13} x {13}	n ≡ {13} x {41}	n ≡ {13} x {83}
n ≡ {17} x {73}	n ≡ {17} x {89}	n ≡ {17} x {31}	n ≡ {17} x {47}	n ≡ {17} x {79}	n ≡ {17} x {37}
n ≡ {19} x {89}	n ≡ {19} x {37}	n ≡ {19} x {23}	n ≡ {19} x {61}	n ≡ {19} x {47}	n ≡ {19} x {71}
n ≡ {23} x {7}	n ≡ {23} x {11}	n ≡ {23} x {19}	n ≡ {23} x {23}	n ≡ {23} x {31}	n ≡ {23} x {43}
n ≡ {29} x {49}	n ≡ {29} x {77}	n ≡ {29} x {43}	n ≡ {29} x {71}	n ≡ {29} x {37}	n ≡ {29} x {31}
n ≡ {31} x {11}	n ≡ {31} x {43}	n ≡ {31} x {17}	n ≡ {31} x {49}	n ≡ {31} x {23}	n ≡ {31} x {29}
n ≡ {37} x {53}	n ≡ {37} x {19}	n ≡ {37} x {41}	n ≡ {37} x {7}	n ≡ {37} x {29}	n ≡ {37} x {17}
n ≡ {41} x {61}	n ≡ {41} x {83}	n ≡ {41} x {37}	n ≡ {41} x {59}	n ≡ {41} x {13}	n ≡ {41} x {79}
n ≡ {43} x {77}	n ≡ {43} x {31}	n ≡ {43} x {29}	n ≡ {43} x {73}	n ≡ {43} x {71}	n ≡ {43} x {23}
n ≡ {47} x {13}	n ≡ {47} x {59}	n ≡ {47} x {61}	n ≡ {47} x {17}	n ≡ {47} x {19}	n ≡ {47} x {67}
n ≡ {49} x {29}	n ≡ {49} x {7}	n ≡ {49} x {53}	n ≡ {49} x {31}	n ≡ {49} x {77}	n ≡ {49} x {11}
n ≡ {53} x {37}	n ≡ {53} x {71}	n ≡ {53} x {49}	n ≡ {53} x {83}	n ≡ {53} x {61}	n ≡ {53} x {73}
n ≡ {59} x {79}	n ≡ {59} x {47}	n ≡ {59} x {73}	n ≡ {59} x {41}	n ≡ {59} x {67}	n ≡ {59} x {61}
n ≡ {61} x {41}	n ≡ {61} x {13}	n ≡ {61} x {47}	n ≡ {61} x {19}	n ≡ {61} x {53}	n ≡ {61} x {59}
n ≡ {67} x {83}	n ≡ {67} x {79}	n ≡ {67} x {71}	n ≡ {67} x {67}	n ≡ {67} x {59}	n ≡ {67} x {47}
n ≡ {71} x {1}	n ≡ {71} x {53}	n ≡ {71} x {67}	n ≡ {71} x {29}	n ≡ {71} x {43}	n ≡ {71} x {19}
n ≡ {73} x {17}	n ≡ {73} x {1}	n ≡ {73} x {59}	n ≡ {73} x {43}	n ≡ {73} x {11}	n ≡ {73} x {53}
n ≡ {77} x {43}	n ≡ {77} x {29}	n ≡ {77} x {1}	n ≡ {77} x {77}	n ≡ {77} x {49}	n ≡ {77} x {7}
n ≡ {79} x {59}	n ≡ {79} x {67}	n ≡ {79} x {83}	n ≡ {79} x {1}	n ≡ {79} x {17}	n ≡ {79} x {41}
n ≡ {83} x {67}	n ≡ {83} x {41}	n ≡ {83} x {79}	n ≡ {83} x {53}	n ≡ {83} x {1}	n ≡ {83} x {13}
n ≡ {89} x {19}	n ≡ {89} x {17}	n ≡ {89} x {13}	n ≡ {89} x {11}	n ≡ {89} x {7}	n ≡ {89} x {1}

Lastly, we conjoin the 36 digital root operations of modulo 30 factorization with the 24 x 24 modulo 90 multiplication matrix to expose the beautiful 'clockworks' at the heart of factorization. The 576 dyads distribute evenly to 36 sets of 16:

From Vedic Square to the Digital Root 'Clockworks' of Modulo 90 Factorization

The 24 Sets of 24 = 576 Modulo 90 Factorization Dyads Organized by their Digital Roots (36 sets of 16 = 576)

for Composite Numbers Not Divisible by 2, 3, or 5 when framed:

n ≡ {1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89} Modulo 90.

Digital root 1 x 1 = 1		Digital root 1 x 2 = 2		Digital root 1 x 4 = 4		Digital root 1 x 5 = 5		Digital root 1 x 7 = 7		Digital root 1 x 8 = 8	
n ≡ 1 (mod 90)	n ≡ 19 (mod 90)	n ≡ 11 (mod 90)	n ≡ 29 (mod 90)	n ≡ 13 (mod 90)	n ≡ 31 (mod 90)	n ≡ 23 (mod 90)	n ≡ 41 (mod 90)	n ≡ 7 (mod 90)	n ≡ 43 (mod 90)	n ≡17 (mod 90)	n ≡ 53 (mod 90)
1 x 1	1 x 19	1 x 11	1 x 29	1 x 13	1 x 31	1 x 23	1 x 41	1 x 7	1 x 43	1 x 17	1 x 53
19 x 19	19 x 1	19 x 29	19 x 11	19 x 67	19 x 49	19 x 77	19 x 59	19 x 43	19 x 7	19 x 53	19 x 17
37 x 73	37 x 37	37 x 83	37 x 47	37 x 49	37 x 13	37 x 59	37 x 23	37 x 61	37 x 79	37 x 71	37 x 89
73 x 37	73 x 73	73 x 47	73 x 83	73 x 31	73 x 67	73 x 41	73 x 77	73 x 79	73 x 61	73 x 89	73 x 71
n ≡ 37 (mod 90)	n ≡ 73 (mod 90)	n ≡ 47 (mod 90)	n ≡ 83 (mod 90)	n ≡ 49 (mod 90)	n ≡ 67 (mod 90)	n ≡ 59 (mod 90)	n ≡ 77 (mod 90)	n ≡ 61 (mod 90)	n ≡ 79 (mod 90)	n ≡ 71 (mod 90)	n ≡ 89 (mod 90)
1 x 37	1 x 73	1 x 47	1 x 83	1 x 49	1 x 67	1 x 59	1 x 77	1 x 61	1 x 79	1 x 71	1 x 89
19 x 73	19 x 37	19 x 83	19 x 47	19 x 31	19 x 13	19 x 41	19 x 23	19 x 79	19 x 61	19 x 89	19 x 71
37 x 1	37 x 19	37 x 11	37 x 29	37 x 67	37 x 31	37 x 77	37 x 41	37 x 43	37 x 7	37 x 53	37 x 17
73 x 19	73 x 1	73 x 29	73 x 11	73 x 13	73 x 49	73 x 23	73 x 59	73 x 7	73 x 43	73 x 17	73 x 53

Digital root 2 x 1 = 2		Digital root 2 x 2 = 4		Digital root 2 x 4 = 8		Digital root 2 x 5 = 1		Digital root 2 x 7 = 5		Digital root 2 x 8 = 7	
n ≡ 11 (mod 90)	n ≡ 29 (mod 90)	n ≡ 13 (mod 90)	n ≡ 31 (mod 90)	n ≡ 17 (mod 90)	n ≡ 53 (mod 90)	n ≡ 1 (mod 90)	n ≡ 19 (mod 90)	n ≡ 23 (mod 90)	n ≡ 41 (mod 90)	n ≡ 7 (mod 90)	n ≡ 43 (mod 90)
11 x 1	11 x 19	11 x 83	11 x 11	11 x 67	11 x 13	11 x 41	11 x 59	11 x 43	11 x 61	11 x 17	11 x 53
29 x 19	29 x 1	29 x 47	29 x 29	29 x 13	29 x 67	29 x 59	29 x 41	29 x 7	29 x 79	29 x 53	29 x 17
47 x 73	47 x 37	47 x 29	47 x 83	47 x 31	47 x 49	47 x 23	47 x 77	47 x 79	47 x 43	47 x 71	47 x 89
83 x 37	83 x 73	83 x 11	83 x 47	83 x 49	83 x 31	83 x 77	83 x 23	83 x 61	83 x 7	83 x 89	83 x 71
n ≡ 47 (mod 90)	n ≡ 83 (mod 90)	n ≡ 49 (mod 90)	n ≡ 67 (mod 90)	n ≡ 71 (mod 90)	n ≡ 89 (mod 90)	n ≡ 37 (mod 90)	n ≡ 73 (mod 90)	n ≡ 59 (mod 90)	n ≡ 77 (mod 90)	n ≡ 61 (mod 90)	n ≡ 79 (mod 90)
11 x 37	11 x 73	11 x 29	11 x 47	11 x 31	11 x 49	11 x 77	11 x 23	11 x 79	11 x 7	11 x 71	11 x 89
29 x 73	29 x 37	29 x 11	29 x 83	29 x 49	29 x 31	29 x 23	29 x 77	29 x 61	29 x 43	29 x 89	29 x 71
47 x 1	47 x 19	47 x 47	47 x 11	47 x 13	47 x 67	47 x 41	47 x 59	47 x 7	47 x 61	47 x 53	47 x 17
83 x 19	83 x 1	83 x 83	83 x 29	83 x 67	83 x 13	83 x 59	83 x 41	83 x 43	83 x 79	83 x 17	83 x 53

Digital root 4 x 1 = 4		Digital root 4 x 2 = 8		Digital root 4 x 4 = 7		Digital root 4 x 5 = 2		Digital root 4 x 7 = 1		Digital root 4 x 8 = 5	
n ≡ 13 (mod 90)	n ≡ 31 (mod 90)	n ≡ 17 (mod 90)	n ≡ 53 (mod 90)	n ≡ 7 (mod 90)	n ≡ 43 (mod 90)	n ≡ 11 (mod 90)	n ≡ 29 (mod 90)	n ≡ 1 (mod 90)	n ≡ 19 (mod 90)	n ≡ 23 (mod 90)	n ≡ 41 (mod 90)
13 x 1	13 x 37	13 x 29	13 x 11	13 x 49	13 x 31	13 x 77	13 x 23	13 x 7	13 x 43	13 x 71	13 x 17
31 x 73	31 x 1	31 x 47	31 x 83	31 x 67	31 x 13	31 x 41	31 x 59	31 x 61	31 x 79	31 x 53	31 x 71
49 x 37	49 x 19	49 x 83	49 x 47	49 x 13	49 x 67	49 x 59	49 x 41	49 x 79	49 x 61	49 x 17	49 x 89
67 x 19	67 x 73	67 x 11	67 x 29	67 x 31	67 x 49	67 x 23	67 x 77	67 x 43	67 x 7	67 x 89	67 x 53
n ≡ 49 (mod 90)	n ≡ 67 (mod 90)	n ≡ 71 (mod 90)	n ≡ 89 (mod 90)	n ≡ 61 (mod 90)	n ≡ 79 (mod 90)	n ≡ 47 (mod 90)	n ≡ 83 (mod 90)	n ≡ 37 (mod 90)	n ≡ 73 (mod 90)	n ≡ 59 (mod 90)	n ≡ 77 (mod 90)
13 x 73	13 x 19	13 x 47	13 x 83	13 x 67	13 x 13	13 x 59	13 x 41	13 x 79	13 x 61	13 x 53	13 x 89
31 x 19	31 x 37	31 x 11	31 x 29	31 x 31	31 x 49	31 x 77	31 x 23	31 x 7	31 x 43	31 x 89	31 x 17
49 x 1	49 x 73	49 x 29	49 x 11	49 x 49	49 x 31	49 x 23	49 x 77	49 x 43	49 x 7	49 x 71	49 x 53
67 x 37	67 x 1	67 x 83	67 x 47	67 x 13	67 x 67	67 x 41	67 x 59	67 x 61	67 x 79	67 x 17	67 x 71

Digital root 5 x 1 = 5		Digital root 5 x 2 = 1		Digital root 5 x 4 = 2		Digital root 5 x 5 = 7		Digital root 5 x 7 = 8		Digital root 5 x 8 = 4	
n ≡ 23 (mod 90)	n ≡ 41 (mod 90)	n ≡ 1 (mod 90)	n ≡ 19 (mod 90)	n ≡ 11 (mod 90)	n ≡ 29 (mod 90)	n ≡ 7 (mod 90)	n ≡ 43 (mod 90)	n ≡ 17 (mod 90)	n ≡ 53 (mod 90)	n ≡ 13 (mod 90)	n ≡ 31 (mod 90)
23 x 1	23 x 37	23 x 47	23 x 83	23 x 67	23 x 13	23 x 59	23 x 41	23 x 79	23 x 61	23 x 71	23 x 17
41 x 73	41 x 1	41 x 11	41 x 29	41 x 31	41 x 49	41 x 77	41 x 23	41 x 7	41 x 43	41 x 53	41 x 71
59 x 37	59 x 19	59 x 29	59 x 11	59 x 49	59 x 31	59 x 23	59 x 77	59 x 43	59 x 7	59 x 17	59 x 89
77 x 19	77 x 73	77 x 83	77 x 47	77 x 13	77 x 67	77 x 41	77 x 59	77 x 61	77 x 79	77 x 89	77 x 53
n ≡ 59 (mod 90)	n ≡ 77 (mod 90)	n ≡ 37 (mod 90)	n ≡ 73 (mod 90)	n ≡ 47 (mod 90)	n ≡ 83 (mod 90)	n ≡ 61 (mod 90)	n ≡ 79 (mod 90)	n ≡ 71 (mod 90)	n ≡ 89 (mod 90)	n ≡ 49 (mod 90)	n ≡ 67 (mod 90)
23 x 73	23 x 19	23 x 29	23 x 11	23 x 49	23 x 31	23 x 77	23 x 23	23 x 7	23 x 43	23 x 53	23 x 89
41 x 19	41 x 37	41 x 47	41 x 83	41 x 67	41 x 13	41 x 41	41 x 59	41 x 61	41 x 79	41 x 89	41 x 17
59 x 1	59 x 73	59 x 83	59 x 47	59 x 13	59 x 67	59 x 59	59 x 41	59 x 79	59 x 61	59 x 71	59 x 53
77 x 37	77 x 1	77 x 11	77 x 29	77 x 31	77 x 49	77 x 23	77 x 77	77 x 43	77 x 7	77 x 17	77 x 71

Digital root 7 x 1 = 7		Digital root 7 x 2 = 5		Digital root 7 x 4 = 1		Digital root 7 x 5 = 8		Digital root 7 x 7 = 4		Digital root 7 x 8 = 2	
n ≡ 7 (mod 90)	n ≡ 43 (mod 90)	n ≡ 23 (mod 90)	n ≡ 41 (mod 90)	n ≡ 1 (mod 90)	n ≡ 19 (mod 90)	n ≡ 17 (mod 90)	n ≡ 53 (mod 90)	n ≡ 13 (mod 90)	n ≡ 31 (mod 90)	n ≡ 11 (mod 90)	n ≡ 29 (mod 90)
7 x 1	7 x 19	7 x 29	7 x 83	7 x 13	7 x 67	7 x 41	7 x 59	7 x 79	7 x 43	7 x 53	7 x 17
43 x 19	43 x 1	43 x 11	43 x 47	43 x 67	43 x 13	43 x 59	43 x 41	43 x 61	43 x 7	43 x 89	43 x 53
61 x 37	61 x 73	61 x 83	61 x 11	61 x 31	61 x 49	61 x 77	61 x 23	61 x 43	61 x 61	61 x 17	61 x 89
79 x 73	79 x 37	79 x 47	79 x 29	79 x 49	79 x 31	79 x 23	79 x 77	79 x 7	79 x 79	79 x 53	79 x 71
n ≡ 61 (mod 90)	n ≡ 79 (mod 90)	n ≡ 59 (mod 90)	n ≡ 77 (mod 90)	n ≡ 37 (mod 90)	n ≡ 73 (mod 90)	n ≡ 71 (mod 90)	n ≡ 89 (mod 90)	n ≡ 49 (mod 90)	n ≡ 67 (mod 90)	n ≡ 47 (mod 90)	n ≡ 83 (mod 90)
7 x 23	7 x 37	7 x 47	7 x 11	7 x 31	7 x 49	7 x 23	7 x 77	7 x 7	7 x 61	7 x 71	7 x 89
43 x 37	43 x 73	43 x 13	43 x 29	43 x 49	43 x 31	43 x 77	43 x 23	43 x 43	43 x 79	43 x 89	43 x 71
61 x 1	61 x 19	61 x 29	61 x 47	61 x 67	61 x 13	61 x 41	61 x 59	61 x 79	61 x 7	61 x 17	61 x 53
79 x 19	79 x 1	79 x 11	79 x 83	79 x 13	79 x 67	79 x 59	79 x 41	79 x 61	79 x 83	79 x 53	79 x 17

Digital root 8 x 1 = 8		Digital root 8 x 2 = 7		Digital root 8 x 4 = 5		Digital root 8 x 5 = 4		Digital root 8 x 7 = 2		Digital root 8 x 8 = 1	
n ≡ 17 (mod 90)	n ≡ 53 (mod 90)	n ≡ 7 (mod 90)	n ≡ 43 (mod 90)	n ≡ 23 (mod 90)	n ≡ 41 (mod 90)	n ≡ 13 (mod 90)	n ≡ 31 (mod 90)	n ≡ 11 (mod 90)	n ≡ 29 (mod 90)	n ≡ 1 (mod 90)	n ≡ 19 (mod 90)
17 x 1	17 x 19	17 x 11	17 x 29	17 x 49	17 x 13	17 x 59	17 x 23	17 x 43	17 x 7	17 x 53	17 x 17
53 x 19	53 x 1	53 x 29	53 x 11	53 x 31	53 x 67	53 x 41	53 x 77	53 x 7	53 x 43	53 x 17	53 x 53
71 x 37	71 x 73	71 x 47	71 x 83	71 x 13	71 x 31	71 x 23	71 x 41	71 x 61	71 x 79	71 x 71	71 x 89
89 x 73	89 x 37	89 x 83	89 x 47	89 x 67	89 x 49	89 x 77	89 x 59	89 x 79	89 x 61	89 x 89	89 x 71
n ≡ 71 (mod 90)	n ≡ 89 (mod 90)	n ≡ 61 (mod 90)	n ≡ 79 (mod 90)	n ≡ 59 (mod 90)	n ≡ 77 (mod 90)	n ≡ 49 (mod 90)	n ≡ 67 (mod 90)	n ≡ 47 (mod 90)	n ≡ 83 (mod 90)	n ≡ 37 (mod 90)	n ≡ 73 (mod 90)
17 x 73	17 x 37	17 x 83	17 x 47	17 x 67	17 x 31	17 x 77	17 x 41	17 x 61	17 x 79	17 x 71	17 x 89
53 x 37	53 x 73	53 x 47	53 x 83	53 x 13	53 x 49	53 x 23	53 x 59	53 x 79	53 x 61	53 x 89	53 x 71
71 x 1	71 x 19	71 x 11	71 x 29	71 x 49	71 x 67	71 x 59	71 x 77	71 x 7	71 x 43	71 x 17	71 x 53
89 x 19	89 x 1	89 x 29	89 x 11	89 x 31	89 x 13	89 x 41	89 x 23	89 x 43	89 x 7	89 x 53	89 x 17

From Vedic Square to the Digital Root 'Clockworks' of Modulo 90 Factorization

These digital root and modulo 90 dyadic patterns expose the profoundly beautiful symmetries and 'initial conditions' that ultimately determine the distribution of *all* prime numbers > 5.

And if you're asking, "How do you account for the first three primes: 2, 3, and 5?" The simple answer is that they and their primorial, 30, determine the *structure* within which factorization algorithms operate. Nor have 3, 6, and 9 disappeared from the scene: They are also present *structurally* as the digital root of modulo 30, 60, 90 cycles, and perhaps more profoundly in the form of rotating symmetry groups: equilateral triangles with vertices {1,4,7} and {2,5,8} that combinatorially rotate the vertices of equilateral triangle {3,6,9}, while the "trinity of triangles" rotate within a 9/3 star polygon in 24 period-24 cycles. $24^2 = 576$.)

Regardless, the beating heart at the center of these patterns is the Vedic Square.

Go figure.

The Guerrilla Arithmetician

primesdemystified.com

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